Problem Set

Problem 1. Compute:

(a) $\int_0^1 \int_x^{x^2} 1\, dy\, dx$

(b) $\iint_R (x+y)\, dA$ where R is the triangular region bounded by $y = 2x, x = 0$ and $y = 4$

(c) $\iint ydA$ where R is the region bounded by $y = x, y = 12 - 2x, y = 0$.

(d) $\int\int_R (4+x^2)\, dA$ where R is the region bounded by the graphs of $y = 1+x^2$ and $y = 9 - x^2$.

(e) $\iint_R (1-y)dA$ where R is the portion of the disk $x^2+y^2 \leq 16$ in the first quadrant

(f) $\int_0^1 \int_y^1 e^{x^2} dxdy$

(g) $\int_1^4 \int_{\sqrt{y}}^2 \sin(\frac{x^3}{3} - x)dxdy$

(h) $\int_0^{\frac{\sqrt{\pi}}{2}} \int_y^{\frac{\sqrt{\pi}}{2}} \cos(x^2)dxdy$

(i) The area of the region bounded by the parabolas $y = x^2$ and $y = \sqrt{x}$ on $[1, 4]$

Problem 2. Compute:

(a) $\iint_R xydA$ where R is the region bounded by the circle $r = 5$.

(b) $\iint_R ydA$ where R is the region bounded by the circle $r = \cos(\theta)$

(c) $\iint_R (x+y)dA$ where R is the region in the first quadrant bounded by $y = 0, y = \sqrt{3}x$ and the circle $r = 2$

(d) $\iint_R x^2dA$ where R is the region bounded by the circle $r = 4\sin(\theta)$

(e) $\iint_R x^2+y^2$ where R is the region inside the circle $r = 2\sin(\theta)$ and outside $r = \sqrt{2}$.

(f) $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2+y^2)dxdy$

(g) $\int_0^1 \int_{\sqrt{x-x^2}}^{\sqrt{1-x^2}} 1dydx$

Problem 3. Compute:

(a) $\iiint_D xydV$ where D is the solid region in the first octant bounded above by the hemisphere $z = \sqrt{4-x^2-y^2}$ and on the sides and bottom by the coordinate planes.

(b) The volume of the solid region bounded by the plane $x + 3y + 6z = 1$ and the three coordinate planes.

(c) The volume of the solid region in the first octant bounded above by the plane $z = 2x$, below by the $xy-$plane and on the sides by the elliptic cylinder $2x^2 + y^2 = 1$ and the plane $y = 0$.

(d) The volume of the solid region which lies above $z = 0$, below the plane $x+y+2z = 4$ and inside the cylinder $x^2 + y^2 = 1$

(e) $\iiint_D x^2 dV$ where D is the solid region between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$

(f) $\iiint_D xyz dV$ where D is the solid region above the cone $z=\sqrt{3x^2+3y^2}$, inside the sphere $x^2+y^2+z^2=3$ and outside $r^2+z^2=1$.

(g) $\iiint_D x^2+y^2 dV$ where D is the solid region bounded above by $x^2+y^2+z^2=4z$, below by the upper nappe of the cone $z^2=x^2+y^2$.

(h) The volume of the solid region bounded above by the upper nappe of the cone $z^2=x^2+y^2$, on the sides by the cylinder $x^2+y^2=4$ and below by the $xy-$plane.

(i) The volume of the solid region above the plane $z=1$ and inside the sphere $x^2+y^2+z^2=2$

Problem 4. An object occupies the solid R bounded below by the paraboloid $z=x^2+y^2$ and above by the plane $z=2$. The mass density of this object at every point is its distance to the origin. Compute the total mass of the object.

Polar Coordinates

In polar coordinates, a point in the plane is represented as:

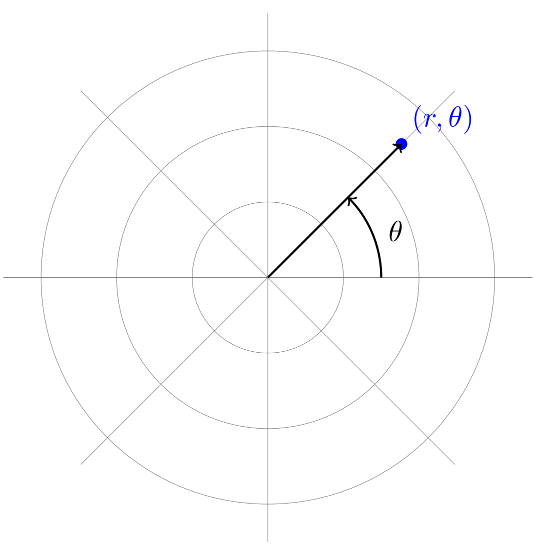
$$(r,\theta)$$

where r is the radial distance from the origin, and θ is the angle from the positive x-axis.

Equations:

$x = r\cos(\theta)$
$y = r\sin(\theta)$
$z = z$.

Useful formulas: $x^2+y^2=r^2$
$\iint_R 1 dxdy = \iint_R r dr d\theta$



Spherical Coordinates

In spherical coordinates, a point in space is represented as:

$$(\rho,\theta,\phi)$$

where ρ is the radial distance from the origin, θ is the angle **of the projected point** from the positive x-axis in the xy-plane, and ϕ is the polar angle from the positive z-axis.

Equations:

$x = \rho\cos(\theta)\sin(\phi)$
$y = \rho\sin(\theta)\sin(\phi)$
$z = \rho\cos(\phi)$.

Useful formulas: $x^2+y^2+z^2=\rho^2$
$x^2+y^2=\rho^2\sin^2(\phi)$
$\iiint_R 1 dxdydz = \iiint_R \rho^2\sin(\phi) d\rho d\phi d\theta$